

D	B	E	C	D	B	D	C	E	A
B	E	C	D	B	D	D	A	E	C
C	D	A	C	E	D	C	C	D	B
D	E	D	D	A	D	D	C	D	C
D	A	B	D	B	D	C	D	C	E
D	B	C	C	E	D	C	C	D	A

...(B) (A)

51	95	70	74	73	90	71	74	90	67
91	72	83	89	50	80	72	84	85	69
62	82	87	76	91	76	87	75	78	79
71	96	81	88	64	82	73	57	86	70
80	81	75	85	74	90	83	66	77	91

70

...

90

80

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.(Quantitative Data)

(Qualitative Data)

:(Qualitative Data)

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:(Quantitative Data)

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.(Frequency Distribution)

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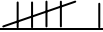

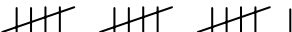


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() : (-)

		()
A		6
B		8
C		16
D		22
E		8
		60

: (-)

() : (-)

	()
A	6
B	8
C	16
D	22
E	8
	60

: (-)

() : (-)

	A	B	C	D	E	
	6	8	16	22	8	60

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(Range)

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$$R = 97 - 50 = 47$$

: R -

$$. 5 = -$$

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: L

$$L = 47 / 5 = 9.4 \sim 10$$

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50 ()

$$. 50 + 10 = 60 :$$

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69

59

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()

:(-)

		()
50-59		3
60-69		5
70-79		18
80-89		16
90-99		8
		50

(-)

:(-)

() : (-)

	()
50-59	3
60-69	5
70-79	18
80-89	16
90-99	8
	50

: (-)

	50-59	60-69	70-79	80-89	90-99	
	3	5	18	16	8	50

: (-)

- .Relative Frequency Table
- .Percentage Frequency Table

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() : (-)

50-59	0.06
60-69	0.10
70-79	0.36
80-89	0.32
90-99	0.16
	1.00

- -

: (-) ()

() : (-)

50-59	6
60-69	10
70-79	36
80-89	32
90-99	16
	100

()

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(-)

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	()
49.5-59.5	3
59.5-69.5	5
69.5-79.5	18
79.5-89.5	16
89.5-99.5	8
	50

: Class Mark

$$/ (+) =$$

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$$, = / (+) = (-)$$

$$, = / (, + ,) = (-)$$

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() : (-)

50-59	49.5-59.5	54.5	3	0.06	6
60-69	59.5-69.5	64.5	5	0.10	10
70-79	69.5-79.5	74.5	18	0.36	36
80-89	79.5-89.5	84.5	16	0.32	32
90-99	89.5-99.5	94.5	8	0.16	16
			50	1.00	100

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Cumulative Frequency "Less Than"

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< 49.5	0
<59.5	3
<69.5	8
<79.5	26
<89.5	42
<99.5	50

Cumulative Frequency "or More"

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= + :

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> 49.5	50
>59.5	47
>69.5	42
>79.5	24
>89.5	8
>99.5	0

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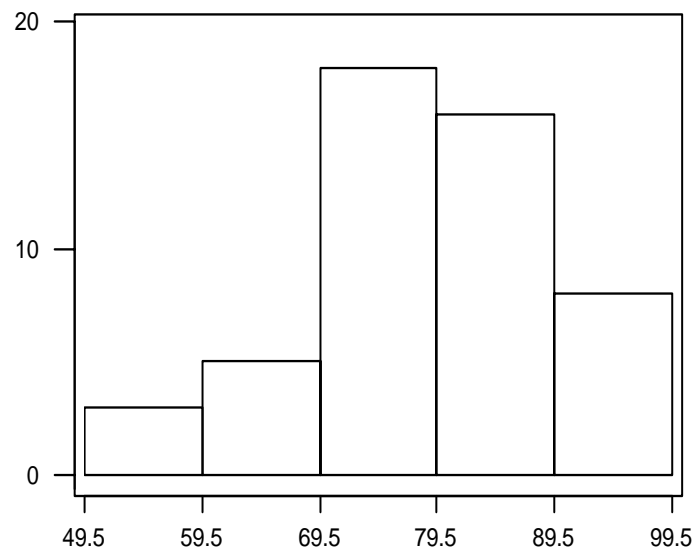
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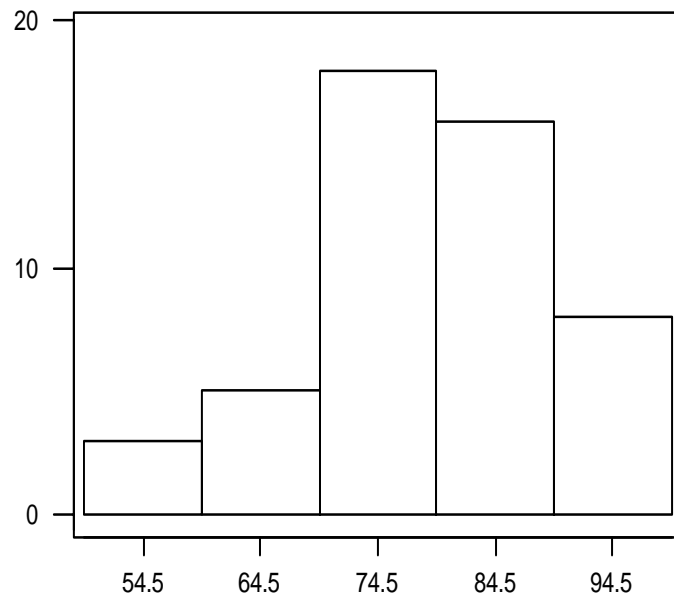


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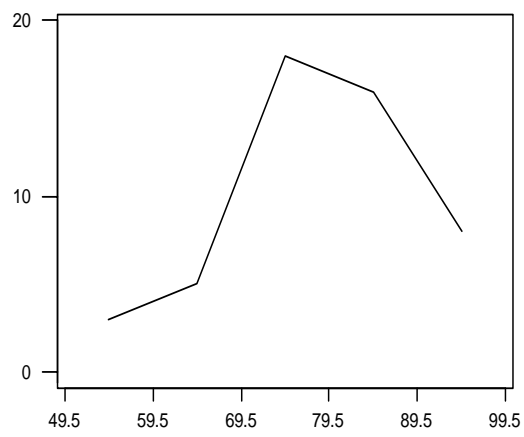
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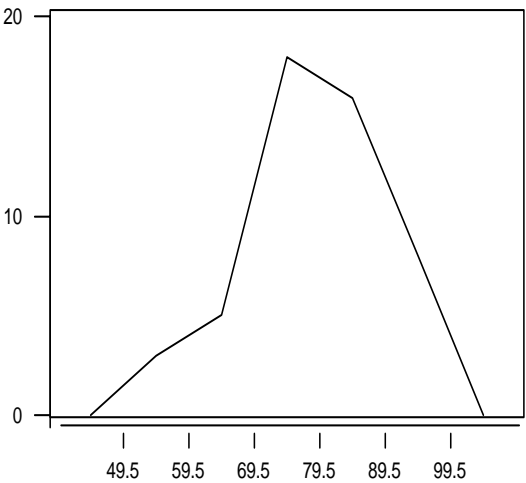
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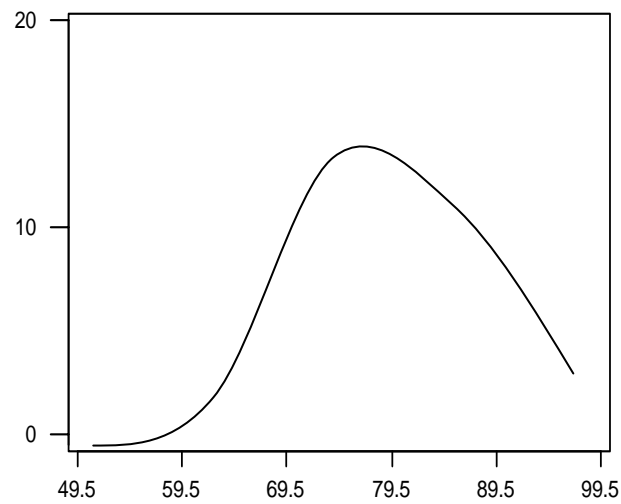
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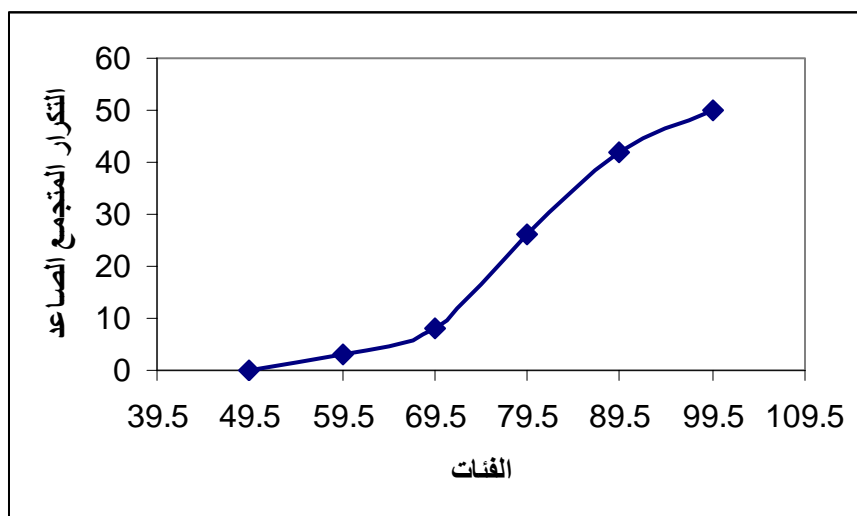
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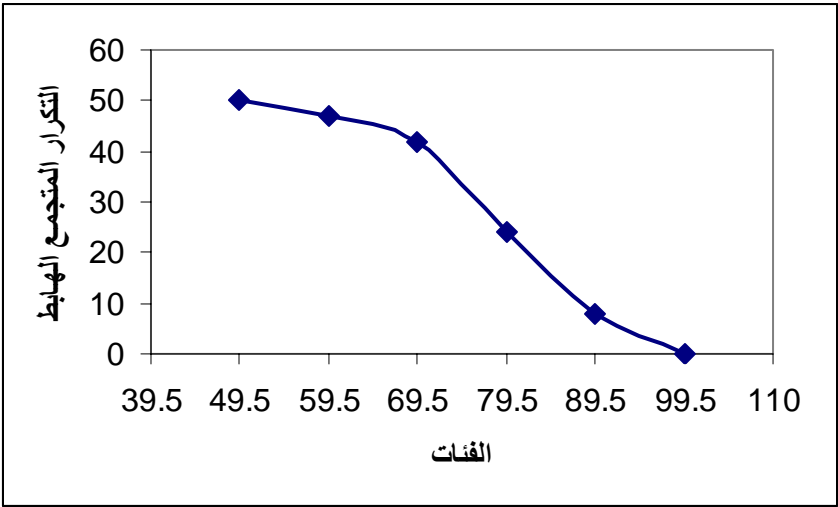
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$$/ (\times) =$$

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	50-69	70-79	80-89	90-99	
	8	18	16	8	50

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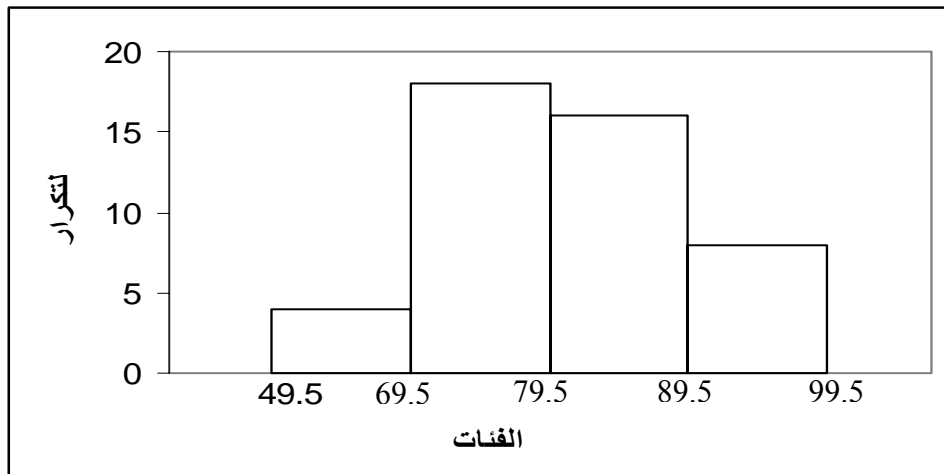
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	49.5-69.5	69.5-79.5	79.5-89.5	89.5-99.5
	8	18	16	8
	4	18	16	8

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132	125	117	124	103	117	110	127	96	129
130	122	118	114	103	119	106	125	114	100
125	128	106	111	115	123	119	114	117	143
136	92	115	118	121	137	139	120	104	125
119	115	101	129	87	108	110	133	135	126
127	103	110	126	118	82	104	137	120	95
146	126	119	105	132	126	118	100	113	119
106	125	117	102	146	129	124	113	95	148

80-89, 90-99, 100-109, ..., 140-149

44	98	40	60	66	71	82	64	72	68
55	69	77	78	88	60	65	68	79	69
62	64	71	66	61	75	83	70	55	62
57	72	61	62	74	62	67	66	60	50

40-49, 50-59, ..., 90-99

0-59	E
60-69	D
70-79	C
80-89	B
90-99	A

	50-59	60-69	70-79	80-89	90-99	100-119	120-129
	8	10	16	15	10	8	3

	155-	158-	161-	164-	167-	170-	173-	176-
	4	10	77	235	368	220	80	6

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 \therefore

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$$\Sigma$$
$$(x_1, x_2, \dots, x_n)$$

•

$$\sum_{i=1}^n x_i = x_1 + x_2 + \dots + x_n$$

$$\bar{x} = \frac{1}{n} \sum_{i=1}^n x_i = \frac{1}{5}(315) = 63$$

:

$$\sum_{i=1}^5 x_i = 63 + 63 + 63 + 63 + 63 = 315$$

.

$$f_1, f_2, \dots, f_k$$

$$(x_1, x_2, \dots, x_k)$$

$$k$$

:

$$\bar{x} = \frac{f_1 x_1 + f_2 x_2 + \dots + f_k x_k}{f_1 + f_2 + \dots + f_k}$$

$$= \frac{\sum_{i=1}^k f_i x_i}{\sum_{i=1}^k f_i}$$

$$= \frac{1}{n} \sum_{i=1}^k f_i x_i$$

:

$$n = \sum_{i=1}^k f_i$$

:

$$\bar{x}$$

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	5-6	7-8	9-10	11-12	13-14
	2	5	8	4	1

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	(x)	(f)	$x f$
5-6	5.5	2	11
7-8	7.5	5	37.5
9-10	9.5	8	76
11-12	11.5	4	46
13-14	13.5	1	13.5
		20	184

$$\bar{x} = \frac{1}{n} \sum_{i=1}^k f_i x_i = \frac{1}{20}(184) = 9.2$$

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$$(d_1, d_2, \dots, d_n)$$

$$(x_1, x_2, \dots, x_n)$$

:

$$d_i = x_i - \bar{x},$$

$$i = 1, 2, \dots, n$$

$$\sum_{i=1}^n (x_i - \bar{x}) = \sum_{i=1}^n d_i = 0$$

:

$$\bar{x} = \frac{1}{n} \sum_{i=1}^n x_i \Rightarrow n \bar{x} = \sum_{i=1}^n x_i \quad \dots (1)$$

$$\therefore \sum_{i=1}^n (x_i - \bar{x}) = \sum_{i=1}^n x_i - \sum_{i=1}^n \bar{x} = \sum_{i=1}^n x_i - n \bar{x} = 0 \quad \dots (2)$$

$$(x_1, x_2, \dots, x_n)$$

 x

:_____

$$: (d_1, d_2, \dots, d_n)$$

 b x

$$d_i = x_i \pm b,$$

$$i = 1, 2, \dots, n$$

:

$$\bar{x} = \bar{d} \mp b$$

:

$$\therefore d_i = x_i \pm b, \quad i = 1, 2, \dots, n$$

$$\Rightarrow \sum_{i=1}^n d_i = \sum_{i=1}^n (x_i \pm b) = \sum_{i=1}^n x_i \pm n b$$

$$\Rightarrow \frac{1}{n} \sum_{i=1}^n d_i = \frac{1}{n} \sum_{i=1}^n x_i \pm b$$

$$\Rightarrow \bar{d} = \bar{x} \pm b$$

$$\Rightarrow \bar{x} = \bar{d} \mp b$$

:

$$\sum_{i=1}^5 d_i$$

$$b = 50$$

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$$\sum_{i=1}^5 d_i = (60 - 50) + (72 - 50) + (40 - 50) + (80 - 50) + (63 - 50)$$

$$= 10 + 22 - 10 + 30 + 13 = 65$$

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$$L$$

$$f_1$$

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< 4.5	0
< 6.5	2
< 8.5	7
< 10.5	15
< 12.5	19
< 14.5	20

15, 7 10 ($\frac{20}{2} = 10$) $\frac{n}{2}$

: 10

$A = 8.5,$ $f_1 = 7,$ $f_2 = 15,$ $L = 10.5 - 8.5 = 2$

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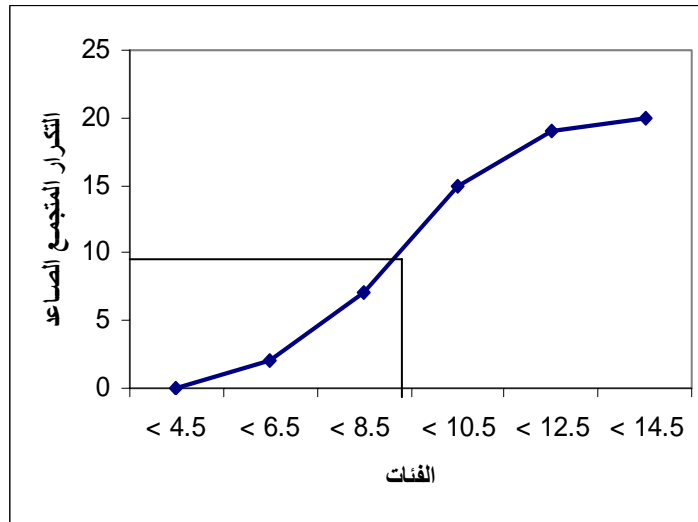
$\text{Med} = 8.5 + \frac{10 - 7}{15 - 7} \cdot 2 = 9.25,$

$\frac{n}{2}$

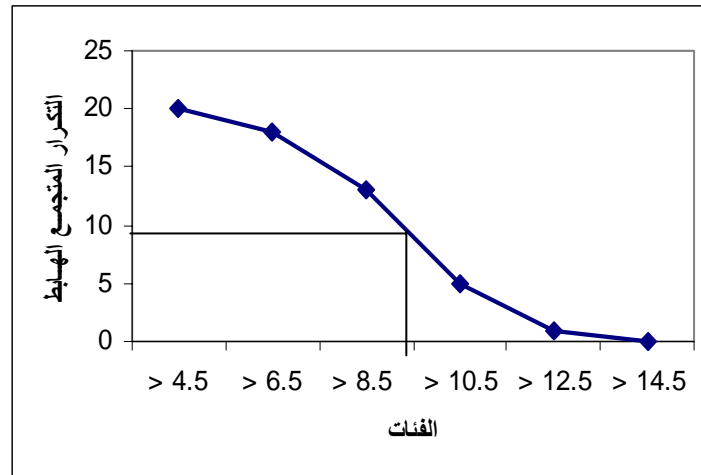
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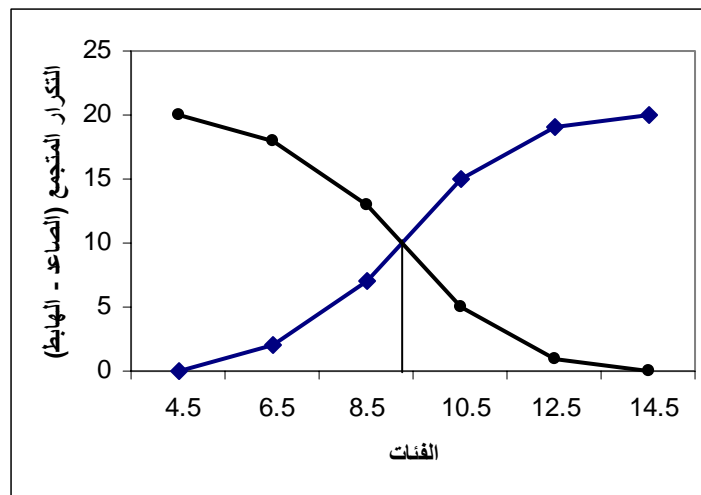
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7 5 :

.5,7

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4, 9, 8, 12, 11, 7, 15 : ()

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$\cdot f_2$ f_1 f -

$\cdot f$ A -

L -

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$$Mod = A + \frac{f - f_1}{2f - f_1 - f_2} \cdot L$$

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	5 – 6	7 – 8	9 – 10	11 – 12	13 – 14
	2	5	8	4	1

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$f = 8,$ $f_1 = 5,$ $f_2 = 4,$ $A = 8.5$
 $L = 10.5 - 8.5 = 2$

:

$Mod = 8.5 + \frac{8 - 5}{16 - 5 - 4} \cdot 2$
 $Mod = 9.36$

$$\left(\frac{3+5+6+6+7+10+12}{7} \right) = 7$$

$$:(\quad)$$

$$3, 5, 6, 6, 7, 10, 12$$

$$:$$

$$\text{G.M.} = \sqrt[7]{3 \cdot 5 \cdot 6 \cdot 6 \cdot 7 \cdot 10 \cdot 12}$$

$$:$$

$$\text{Log G.M.} = \frac{1}{7} (\text{Log } 3 + \text{Log } 5 + \text{Log } 6 + \text{Log } 6 + \text{Log } 7 + \text{Log } 10 + \text{Log } 12)$$

$$= \frac{1}{7} (0.4771 + 0.699 + 0.7782 + 0.7782 + 0.8451 + 1 + 1.0729)$$

$$= 0.8081$$

$$\text{G.M.} = 6.43$$

$$\bar{x} = \frac{1}{7} (3 + 5 + 6 + 6 + 7 + 10 + 12) = 7$$

$$\text{G.M.}$$

$$\bar{x}$$

$$.$$

$$\left(\frac{3+5+6+6+7+10+12}{7} \right)$$

$$x_1, x_2, \dots, x_k$$

$$k$$

$$:$$

$$f_1, f_2, \dots, f_k$$

$$\text{G.M.} = \sqrt[n]{x_1^{f_1} x_2^{f_2} \dots x_n^{f_n}}$$

$$n = \sum_{i=1}^k f_i :$$

$$-$$

$$x_1, x_2, \dots, x_n$$

$$H$$

$$.$$

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$$H = \frac{1}{\frac{1}{n} \sum_{i=1}^n \frac{1}{x_i}} = \frac{n}{\sum_{i=1}^n \frac{1}{x_i}}$$

$$:$$

$$\frac{1}{H} = \frac{1}{n} \sum_{i=1}^n \frac{1}{x_i}$$

$$\frac{2n}{4}$$

$$\frac{n}{4}$$

$$\frac{n}{2}$$

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$$\frac{2n}{100}$$

$$\frac{n}{100}$$

$$\frac{n}{2}$$

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:

< 4.5	0
< 6.5	2
D ₂	
< 8.5	7
< 10.5	15
P ₉₀	
< 12.5	19
< 14.5	20

:

D₂

$$D_2 = A + \frac{(\frac{2n}{10} - f_1)}{f_2 - f_1} L$$

$$f_2$$

$$f_1$$

n

A

:

L

$$D_2 = 6.5 + \frac{4-2}{7-2}.2 = 7.3$$

:

P₉₀

$$P_{90} = A' + \frac{(\frac{90n}{100} - f'_1)}{f'_2 - f'_1} L$$

$$P_{90} = 10.5 + \frac{18-15}{19-15}.2 = 12$$

$$\frac{y}{x} = -$$

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Z Y X

X	59,61,62,58,60
Y	50,60,66,54,70
Z	39,65,46,78,72

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.Measures of Dispersion

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n

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Qn Q_2 Qn

$\frac{n}{4}$

Q_1

$\frac{3n}{4}$

Q_3

Q_3 Q_1

Q

:

$$Q = \frac{Q_3 - Q_1}{2}, \tag{1}$$

Q_2 $\frac{n}{2}$

Q_2

Q

:

-

Q_1

-

Q_3

-

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$$\vdots \qquad \qquad \qquad \vdots (\quad)$$

67,65,69,58,55,71,72,70

55,58,65,67,69,70,71,72

$$Q_1 = \frac{58+65}{2} = 61.5,$$

$$Q_3 = \frac{70 + 71}{2} = 70.5$$

$$Q = \frac{Q_3 - Q_1}{2} = 4.5$$

$$: \qquad \qquad \qquad : (\)$$

59,67,65,69,58,55,70,72,74

55,58,59,65,67,69,70,72,74

$$Q_1 = 59,$$

$$Q_3 = 70$$

$$Q = \frac{Q_3 - Q_1}{2} = 5.5$$

$$\frac{n}{2} \quad \frac{n}{4} \quad Q_1$$

$$\frac{n}{2} \quad \frac{3n}{4} \quad Q_3$$

$$\vdots \quad Q_3 \quad Q_1 \quad ()$$

$$Q_1 = A_1 + \frac{(\frac{n}{4} - f_1)}{f_2 - f_1} L, \quad (2)$$

$$Q_3 = A_2 + \frac{(\frac{3n}{4} - f'_1)}{f'_2 - f'_1} L, \quad (3)$$

$$\cdot(\cdot) \qquad \qquad \qquad :(\cdot)$$

< 39.5	0
< 49.5	2
< 59.5	11
< 69.5	26
< 79.5	37
< 89.5	39
< 99.5	40

$$n = 40, \quad \frac{n}{4} = 10, \quad \frac{3n}{4} = 30, \quad L = 10$$

$$Q_1 = 49.5 + \left(\frac{10 - 2}{11 - 2} \right) 10$$

$$Q_1 = 49.5 + 8.89 = 58.39$$

$$Q_3 = 69.5 + \left(\frac{30 - 26}{37 - 26} \right) 10$$

$$Q_3 = 69.5 + 3.64 = 73.14$$

$$Q = \frac{Q_3 - Q_1}{2} = 7.38$$

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 \bar{x}

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M.D.

$$\text{M.D.} = \frac{1}{n} \sum_{i=1}^n |x_i - \bar{x}|, \quad (4)$$

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$$\bar{x} = \frac{\sum fx}{n}$$
$$\bar{x} = \frac{2630}{40} = 65.75$$
$$\text{M.D.} = \frac{325}{40} = 8.125,$$

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σ^2

σ

$$\vdots$$

$$N$$

$$\overline{X}$$

$$X_1, X_2, \dots, X_N$$

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$$(X_1 - \bar{X})^2, (X_2 - \bar{X})^2, \dots, (X_N - \bar{X})^2$$

$$S^2 = \frac{1}{5-1}(10) = 2.5$$

$$S = \sqrt{2.5} = 1.581,$$

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$$S^2 = \frac{1}{n-1} \left(\sum_{i=1}^n x_i^2 - \frac{\left(\sum_{i=1}^n x_i \right)^2}{n} \right), \quad (10)$$

:

$$S^2 = \frac{1}{n-1} \sum_{i=1}^n (x_i - \bar{x})^2$$

$$S^2 = \frac{1}{n-1} \sum_{i=1}^n (x_i^2 - 2x_i\bar{x} + \bar{x}^2)$$

$$S^2 = \frac{1}{n-1} \left(\sum_{i=1}^n x_i^2 - 2\bar{x} \sum_{i=1}^n x_i + n\bar{x}^2 \right)$$

$$S^2 = \frac{1}{n-1} \left(\sum_{i=1}^n x_i^2 - n\bar{x}^2 \right)$$

$$S^2 = \frac{1}{n-1} \left(\sum_{i=1}^n x_i^2 - \frac{\left(\sum_{i=1}^n x_i \right)^2}{n} \right)$$

$$. \quad \sum x^2 \quad \sum x \quad ()$$

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x	x^2
8	64
9	81
7	49
6	36
5	25
35	255

$$S^2 = \frac{1}{n-1} \left(\sum_{i=1}^n x_i^2 - \frac{\left(\sum_{i=1}^n x_i \right)^2}{n} \right)$$

$$S^2 = \frac{1}{4} \left(255 - \frac{(35)^2}{5} \right) = 2.5$$

$$S = \sqrt{2.5} = 1.581,$$

()

$$(n_1 - 1) S_1^2 = \sum_{i=1}^{n_1} (x_i - \bar{x})^2$$

$$(n_2 - 1) S_2^2 = \sum_{i=1}^{n_2} (y_i - \bar{y})^2$$

$$(n_1 - 1) S_1^2 + (n_2 - 1) S_2^2 = \sum_{i=1}^{n_1} (x_i - \bar{x})^2 + \sum_{i=1}^{n_2} (y_i - \bar{y})^2 = \sum_{i=1}^{n_1+n_2} (z_i - \bar{x})^2$$

$$\therefore S^2 = \frac{(n_1 - 1)S_1^2 + (n_2 - 1)S_2^2}{n_1 + n_2 - 1}$$

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 f_1, f_2, \dots, f_k x_1, x_2, \dots, x_k k

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$$S^2 = \frac{1}{n-1} \sum_{i=1}^n f_i (x_i - \bar{x})^2, \quad S = \sqrt{\frac{1}{n-1} \sum_{i=1}^n f_i (x_i - \bar{x})^2}, \quad (14)$$

$$S^2 = \frac{1}{n-1} \left(\sum_{i=1}^n f_i x_i^2 - \frac{(\sum_{i=1}^n f_i x_i)^2}{n} \right), \quad (15)$$

$$S^2 = \frac{1}{n-1} \left(\sum_{i=1}^n f_i d_i^2 - \frac{(\sum_{i=1}^n f_i d_i)^2}{n} \right), \quad (16)$$

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Classes	x	f	xf	$x - \bar{x}$	$(x - \bar{x})^2$	$(x - \bar{x})^2 f$
40-49	44.5	2	89	-21.25	451.56	903.13
50-59	55.5	9	490.5	-11.25	126.56	1139.06
60-69	65.5	15	967.5	-1.25	1.56	23.44
70-79	75.5	11	819.5	8.75	76.56	842.19
80-89	85.5	2	169	18.75	351.56	703.13
90-99	95.5	1	94.5	28.75	826.56	826.56
Total		40	2630			4437.5

$$\bar{x} = \frac{1}{n} \sum_{i=1}^k f_i x_i = \frac{1}{40} (2630) = 65.75$$

$$S^2 = \frac{1}{n-1} \sum_{i=1}^n f_i (x_i - \bar{x})^2 = \frac{1}{40-1} (4437.5) = 113.78$$

$$S = 10.67,$$

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Classes	x	f	xf	$x^2 f$
40-49	44.5	2	89	3960.5
50-59	55.5	9	490.5	26732.25
60-69	65.5	15	967.5	62403.75
70-79	75.5	11	819.5	61052.75
80-89	85.5	2	169	12280.5
90-99	95.5	1	94.5	8930.25
Total		40	2630	177360

$$S^2 = \frac{1}{n-1} \left(\sum_{i=1}^n f_i x_i^2 - \frac{(\sum_{i=1}^n f_i x_i)^2}{n} \right) = \frac{1}{40-1} (177360 - 172922.5) = 113.78$$

$$S = 10.67, \quad ()$$

$$c = 64.5 \quad () \quad ()$$

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Classes	x	f	$d = x - 64.5$	df	$d^2 f$
40-49	44.5	2	-20	-40	800
50-59	55.5	9	-10	-90	900
60-69	65.5	15	0	0	0
70-79	75.5	11	10	110	1100
80-89	85.5	2	20	40	800
90-99	95.5	1	30	30	900
Total		40		50	4500

$$S^2 = \frac{1}{n-1} \left(\sum_{i=1}^n f_i d_i^2 - \frac{(\sum_{i=1}^n f_i d_i)^2}{n} \right) = \frac{1}{40-1} (4500 - 62.5) = 113.78$$

$$S = 10.67,$$

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Classes	x	f	$d = x - 64.5$	$d/10 = d'$	$d'f$	$d'^2 f$
40-49	44.5	2	-20	-2	-4	8
50-59	55.5	9	-10	-1	-9	9
60-69	65.5	15	0	0	0	0
70-79	75.5	11	10	1	11	11
80-89	85.5	2	20	2	4	8
90-99	95.5	1	30	3	3	9
Total		40			5	45

$$S_{d'}^2 = \frac{1}{n-1} \left(\sum_{i=1}^n f_i d_i'^2 - \frac{(\sum_{i=1}^n f_i d_i')^2}{n} \right) = \frac{1}{40-1} (45 - 0.625) = 1.1378$$

$$S_{d'}=1.067, \quad S_x=10 S_{d'}=10.67 \quad ($$

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$$\overline{x}$$

$$x_1, x_2, \dots, x_n$$

X

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$$\mathbf{Z}$$

S

$$Z_i = \frac{x_i - \bar{x}}{s}, \quad i = 1, 2, \dots, n$$

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$$\mathbf{Z}_i$$

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$$Z_1$$

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$$Z_1 = \frac{82 - 75}{10} = 0.7$$

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Z_2

$$Z_2 = \frac{89 - 81}{16} = 0.5$$

$$1- r = \frac{3(\bar{x} - Med)}{s}, \quad (17)$$

$$2- r = \frac{\bar{x} - Mod}{s}, \quad (18)$$

$$3- r = \frac{m_3^2}{s^3}, \quad (19)$$

$$m_3 = \frac{\sum (x - \bar{x})^3}{n},$$

$$m_3 = \frac{\sum f(x - \bar{x})^3}{n},$$

$$\bar{x} = 65.75, \quad Med = 65.41, \quad Mod = 65.5, \quad s = 10.67$$

$$r = \frac{3(\bar{x} - Med)}{s} = \frac{3(65.75 - 65.41)}{10.67} = 0.069, \quad (1)$$

$$r = \frac{\bar{x} - Mod}{s} = \frac{65.75 - 65.5}{10.67} = 0.025, \quad (2)$$

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$$k = \frac{m_4}{s^4} - 3$$
$$m_4 = \frac{\sum (x - \bar{x})^4}{n},$$

$$m_4 = \frac{\sum f(x - \bar{x})^4}{n},$$

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6,3,5,5,9,4,6,7,1,2,4,8

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	58-60	61-63	64-66	67-69	70-72	73-75
	2	7	14	15	8	4

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	< 10	10-14	15-19	20-24	25-29	30≤
	5	20	35	19	13	8

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2, 5, 9, 4, 3, 6

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70, 70, 70, 70, 70, 70, 70

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